

Calculation Policy: Y6

Mathematical Manipulatives | Key Representations
Progression in Procedures



Avonwood Primary School

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Key vocabulary

Place value: ones, tens, hundreds, column, ascending, descending, consecutive

Addition: sum, addend, add

Subtraction: difference, subtrahend, subtract

Multiplication: product, multiplicand, multiplier, multiply, repeated addition, composite number, multiple, product

Division: quotient, dividend, divisor, divide, repeated subtraction, bisect, factor

Fractions: denominator, numerator, equal part, whole

Manipulatives: place value counters, Dienes

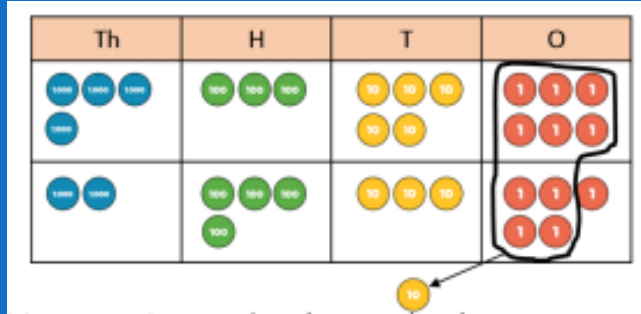
Representations: represent, representation, numberline, array, row/column, Part-Part-Whole diagram, bar model



YEAR 6: Addition

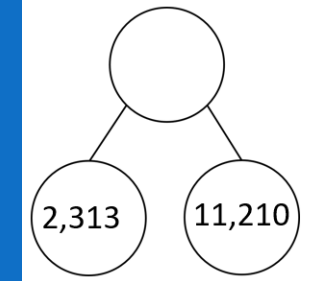
Manipulatives

The recommended manipulatives (physical resources) for adding numbers with more than 4- digits are **place value counters and Dienes**. This should build on prior knowledge.



Representations

The key representations used are **place value grids, bar models and part-part-whole diagrams** (which encourage children to apply their knowledge of place value).



Factual knowledge

The key factual knowledge includes recall of addition/subtraction facts to 20, doubling/halving facts to 20 and recall of 2, 3, 4, 5, 8 and 10 multiplication tables.

Addition Tables					
ones	tens	hundreds	thousands	ten thousands	hundred thousands
1+0=1	2+0=2	3+0=3	4+0=4	5+0=5	6+0=6
1+1=2	2+1=3	3+1=4	4+1=5	5+1=6	6+1=7
1+2=3	2+2=4	3+2=5	4+2=6	5+2=7	6+2=8
1+3=4	2+3=5	3+3=6	4+3=7	5+3=8	6+3=9
1+4=5	2+4=6	3+4=7	4+4=8	5+4=9	6+4=10
1+5=6	2+5=7	3+5=8	4+5=9	5+5=10	6+5=11
1+6=7	2+6=8	3+6=9	4+6=10	5+6=11	6+6=12
1+7=8	2+7=9	3+7=10	4+7=11	5+7=12	6+7=13
1+8=9	2+8=10	3+8=11	4+8=12	5+8=13	6+8=14
1+9=10	2+9=11	3+9=12	4+9=13	5+9=14	6+9=15
1+10=11	2+10=12	3+10=13	4+10=14	5+10=15	6+10=16
1+11=12	2+11=13	3+11=14	4+11=15	5+11=16	6+11=17
1+12=13	2+12=14	3+12=15	4+12=16	5+12=17	6+12=18
2+0=2	3+0=3	4+0=4	5+0=5	6+0=6	7+0=7
2+1=3	3+1=4	4+1=5	5+1=6	6+1=7	7+1=8
2+2=4	3+2=5	4+2=6	5+2=7	6+2=8	7+2=9
2+3=5	3+3=6	4+3=7	5+3=8	6+3=9	7+3=10
2+4=6	3+4=7	4+4=8	5+4=9	6+4=10	7+4=11
2+5=7	3+5=8	4+5=9	5+5=10	6+5=11	7+5=12
2+6=8	3+6=9	4+6=10	5+6=11	6+6=12	7+6=13
2+7=9	3+7=10	4+7=11	5+7=12	6+7=13	7+7=14
2+8=10	3+8=11	4+8=12	5+8=13	6+8=14	7+8=15
2+9=11	3+9=12	4+9=13	5+9=14	6+9=15	7+9=16
2+10=12	3+10=13	4+10=14	5+10=15	6+10=16	7+10=17
2+11=13	3+11=14	4+11=15	5+11=16	6+11=17	7+11=18
2+12=14	3+12=15	4+12=16	5+12=17	6+12=18	7+12=19
3+0=3	4+0=4	5+0=5	6+0=6	7+0=7	8+0=8
3+1=4	4+1=5	5+1=6	6+1=7	7+1=8	8+1=9
3+2=5	4+2=6	5+2=7	6+2=8	7+2=9	8+2=10
3+3=6	4+3=7	5+3=8	6+3=9	7+3=10	8+3=11
3+4=7	4+4=8	5+4=9	6+4=10	7+4=11	8+4=12
3+5=8	4+5=9	5+5=10	6+5=11	7+5=12	8+5=13
3+6=9	4+6=10	5+6=11	6+6=12	7+6=13	8+6=14
3+7=10	4+7=11	5+7=12	6+7=13	7+7=14	8+7=15
3+8=11	4+8=12	5+8=13	6+8=14	7+8=15	8+8=16
3+9=12	4+9=13	5+9=14	6+9=15	7+9=16	8+9=17
3+10=13	4+10=14	5+10=15	6+10=16	7+10=17	8+10=18
3+11=14	4+11=15	5+11=16	6+11=17	7+11=18	8+11=19
3+12=15	4+12=16	5+12=17	6+12=18	7+12=19	8+12=20
4+0=4	5+0=5	6+0=6	7+0=7	8+0=8	9+0=9
4+1=5	5+1=6	6+1=7	7+1=8	8+1=9	9+1=10
4+2=6	5+2=7	6+2=8	7+2=9	8+2=10	9+2=11
4+3=7	5+3=8	6+3=9	7+3=10	8+3=11	9+3=12
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4+5=9	5+5=10	6+5=11	7+5=12	8+5=13	9+5=14
4+6=10	5+6=11	6+6=12	7+6=13	8+6=14	9+6=15
4+7=11	5+7=12	6+7=13	7+7=14	8+7=15	9+7=16
4+8=12	5+8=13	6+8=14	7+8=15	8+8=16	9+8=17
4+9=13	5+9=14	6+9=15	7+9=16	8+9=17	9+9=18
4+10=14	5+10=15	6+10=16	7+10=17	8+10=18	9+10=19
4+11=15	5+11=16	6+11=17	7+11=18	8+11=19	9+11=20
4+12=16	5+12=17	6+12=18	7+12=19	8+12=20	9+12=21
5+0=5	6+0=6	7+0=7	8+0=8	9+0=9	10+0=10
5+1=6	6+1=7	7+1=8	8+1=9	9+1=10	10+1=11
5+2=7	6+2=8	7+2=9	8+2=10	9+2=11	10+2=12
5+3=8	6+3=9	7+3=10	8+3=11	9+3=12	10+3=13
5+4=9	6+4=10	7+4=11	8+4=12	9+4=13	10+4=14
5+5=10	6+5=11	7+5=12	8+5=13	9+5=14	10+5=15
5+6=11	6+6=12	7+6=13	8+6=14	9+6=15	10+6=16
5+7=12	6+7=13	7+7=14	8+7=15	9+7=16	10+7=17
5+8=13	6+8=14	7+8=15	8+8=16	9+8=17	10+8=18
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5+10=15	6+10=16	7+10=17	8+10=18	9+10=19	10+10=20
5+11=16	6+11=17	7+11=18	8+11=19	9+11=20	10+11=21
5+12=17	6+12=18	7+12=19	8+12=20	9+12=21	10+12=22
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6+7=13	7+7=14	8+7=15	9+7=16	10+7=17	11+7=18
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6+11=17	7+11=18	8+11=19	9+11=20	10+11=21	11+11=22
6+12=18	7+12=19	8+12=20	9+12=21	10+12=22	11+12=23
7+0=7	8+0=8	9+0=9	10+0=10	11+0=11	12+0=12
7+1=8	8+1=9	9+1=10	10+1=11	11+1=12	12+1=13
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7+10=17	8+10=18	9+10=19	10+10=20	11+10=21	12+10=22
7+11=18	8+11=19	9+11=20	10+11=21	11+11=22	12+11=23
7+12=19	8+12=20	9+12=21	10+12=22	11+12=23	12+12=24

Procedural knowledge

The key methods is **formal column addition**.

	Th	H	T	O
	4	3	5	6
+	2	4	3	5
	6	7	9	1
				1

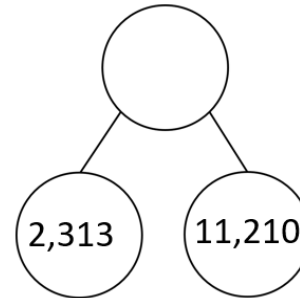
Addition in Year 6

1. The recommended manipulatives (physical resources) for adding numbers with more than 4-digits are **place value counters and dienes**.
2. The key representations used are: **part-part-whole diagrams and , bar models** (which encourage children to apply their knowledge of place value) and **place value grids**.
3. The key methods is **formal column addition**.

1

Th	H	T	O
●●●●	●●●●	●●●●	●●●●
●●	●●●●	●●●●	●●●●

2



3

	Th	H	T	O
	4	3	5	6
+	2	4	3	5
	6	7	9	1

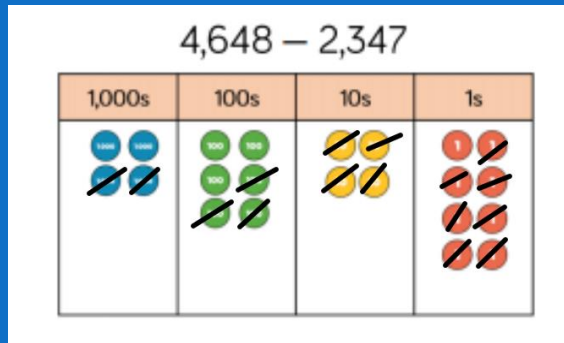
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YEAR 6: Subtraction

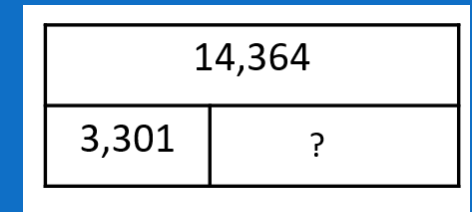
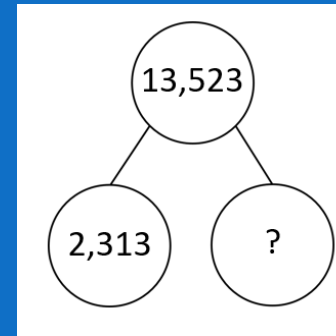
Manipulatives

The recommended manipulatives (physical resources) for subtracting numbers more than 4-digits are **place value counters and Dienes**.



Representations

The key representations used are **place value grids, bar models and part-part-whole diagrams** (which encourage children to apply their knowledge of place value).



Factual knowledge

The key factual knowledge includes recall of addition/subtraction facts to 20, doubling/halving facts to 20 and recall of 2, 3, 4, 5, 8 and 10 multiplication tables.

Subtraction Tables					
Subtracting 1	Subtracting 2	Subtracting 3	Subtracting 4	Subtracting 5	Subtracting 6
10-1	11-2	12-3	13-4	14-5	15-6
9-1	10-2	11-3	12-4	13-5	14-6
8-1	9-2	10-3	11-4	12-5	13-6
7-1	8-2	9-3	10-4	11-5	12-6
6-1	7-2	8-3	9-4	10-5	11-6
5-1	6-2	7-3	8-4	9-5	10-6
4-1	5-2	6-3	7-4	8-5	9-6
3-1	4-2	5-3	6-4	7-5	8-6
2-1	3-2	4-3	5-4	6-5	7-6

Procedural knowledge

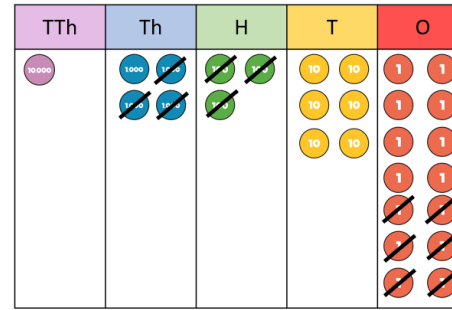
The key methods is **formal column subtraction**.

TTh	Th	H	T	O
			5	1
1	4	3	6	4
	3	3	0	6
1	1	0	5	8

Subtraction in Year 6

1. The recommended manipulatives (physical resources) for subtracting numbers with more than 4- digits are **place value counters and dienes**.
2. The key representations used are: **part-part-whole diagrams, bar models** (which encourage children to apply their knowledge of place value) and **place value grids**.
3. The key methods is **formal column subtraction**.

1



2

14,364	
3,301	?

3

TTh	Th	H	T	O
1	4	3	6 ⁵	4 ¹
	3	3	0	6
1	1	0	5	8

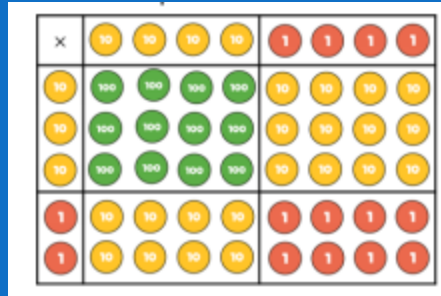


YEAR 6: Multiplication

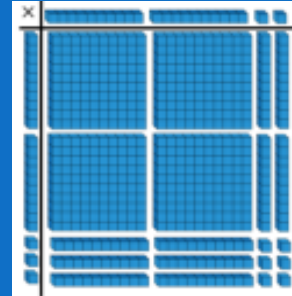
Manipulatives

The recommended manipulatives (physical resources) for multiplying numbers with up to 4- digits are **place value counters** and **Dienes**.

$44 \times 32 =$

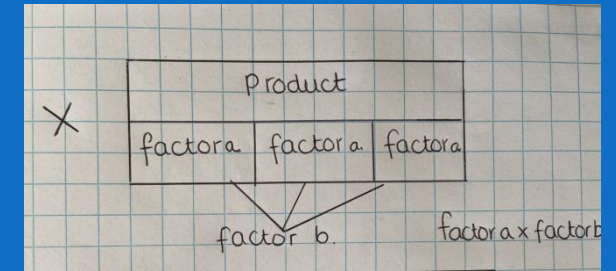


$23 \times 22 =$



Representations

The key representations used are **place value grids** and **bar models**.



Factual knowledge

The key factual knowledge includes recall of addition/subtraction facts to 20, doubling/halving facts to 20 and recall of multiplication tables to 12.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

1x	2x	3x	4x	5x	6x
1x1=1	1x2=2	1x3=3	1x4=4	1x5=5	1x6=6
2x1=2	2x2=4	2x3=6	2x4=8	2x5=10	2x6=12
3x1=3	3x2=6	3x3=9	3x4=12	3x5=15	3x6=18
4x1=4	4x2=8	4x3=12	4x4=16	4x5=20	4x6=24
5x1=5	5x2=10	5x3=15	5x4=20	5x5=25	5x6=30
6x1=6	6x2=12	6x3=18	6x4=24	6x5=30	6x6=36
7x1=7	7x2=14	7x3=21	7x4=28	7x5=35	7x6=42
8x1=8	8x2=16	8x3=24	8x4=32	8x5=40	8x6=48
9x1=9	9x2=18	9x3=27	9x4=36	9x5=45	9x6=54
10x1=10	10x2=20	10x3=30	10x4=40	10x5=50	10x6=60
11x1=11	11x2=22	11x3=33	11x4=44	11x5=55	11x6=66
12x1=12	12x2=24	12x3=36	12x4=48	12x5=60	12x6=72
7x	8x	9x	10x	11x	12x
1x7=7	1x8=8	1x9=9	1x10=10	1x11=11	1x12=12
2x7=14	2x8=16	2x9=18	2x10=20	2x11=22	2x12=24
3x7=21	3x8=24	3x9=27	3x10=30	3x11=33	3x12=36
4x7=28	4x8=32	4x9=36	4x10=40	4x11=44	4x12=48
5x7=35	5x8=40	5x9=45	5x10=50	5x11=55	5x12=60
6x7=42	6x8=48	6x9=54	6x10=60	6x11=66	6x12=72
7x7=49	7x8=56	7x9=63	7x10=70	7x11=77	7x12=84
8x7=56	8x8=64	8x9=72	8x10=80	8x11=88	8x12=96
9x7=63	9x8=72	9x9=81	9x10=90	9x11=99	9x12=108
10x7=70	10x8=80	10x9=90	10x10=100	10x11=110	10x12=120
11x7=77	11x8=88	11x9=99	11x10=110	11x11=121	11x12=132
12x7=84	12x8=96	12x9=108	12x10=120	12x11=132	12x12=144

Procedural knowledge

The key methods are **grid method** and **formal column multiplication**.

x	40	4
30	1,200	120
2	80	8

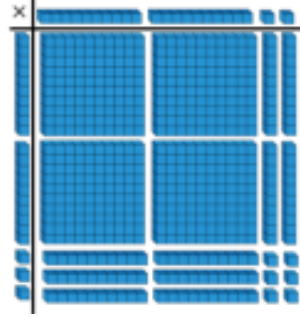
	TTh	Th	H	T	O
1,207	1	2	0	7	
x				3	6
	7	2	4	2	
+	3	6	2	1	0
	4	3	4	5	2
					1

Multiplication in Year 6

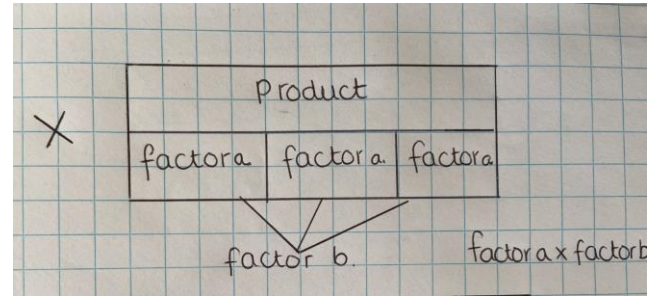
1. The recommended manipulatives (physical resources) for multiplying 4- digit numbers by 2- digit numbers are **place value counters and dienes**.
2. The key representations used are: **bar model** and **place value grids**.
3. The key methods are **Grid method**, and **formal column method**.

1

$$22 \times 23 = 506$$



2



3

x	40	4
30	1,200	120
2	80	8

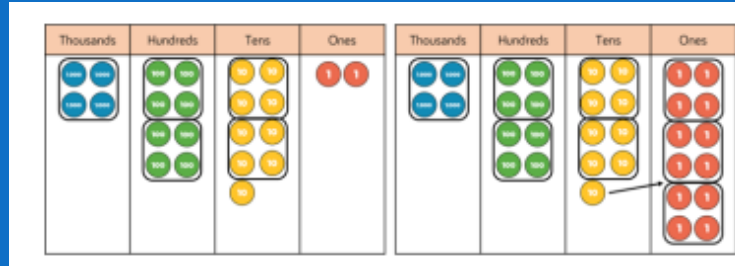
		1,207 × 36					
		TTh	Th	H	T	O	
		1	2	0	7		
x				3	6		
		7	2	4	2	(1,207 × 6)	
	+	3	6	2	1	0	(1,207 × 30)
		4	3	4	5	2	
		1					



YEAR 6: Division

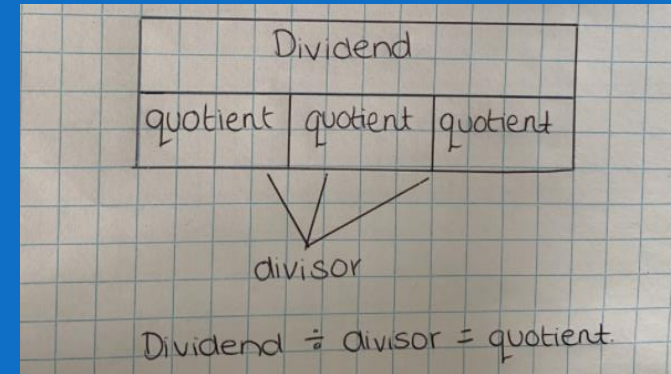
Manipulatives

The recommended manipulatives (physical resources) for Division numbers with up are **place value counters and Dienes**.



Representations

The key representations used are **bar models**.



Factual knowledge

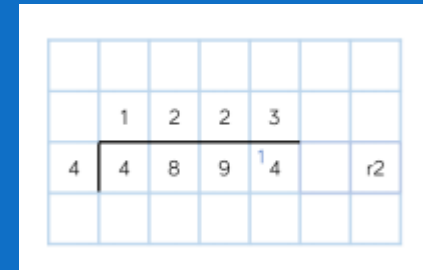
The key factual knowledge includes recall of addition/subtraction facts to 20, doubling/halving facts to 20 and recall of 2, 3, 4, 5, 8 and 10 multiplication tables.

	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12
=1	1	2	3	4	5	6	7	8	9	10	11	12
=2	2	4	6	8	10	12	14	16	18	20	22	24
=3	3	6	9	12	15	18	21	24	27	30	33	36
=4	4	8	12	16	20	24	28	32	36	40	44	48
=5	5	10	15	20	25	30	35	40	45	50	55	60
=6	6	12	18	24	30	36	42	48	54	60	66	72
=7	7	14	21	28	35	42	49	56	63	70	77	84
=8	8	16	24	32	40	48	56	64	72	80	88	96
=9	9	18	27	36	45	54	63	72	81	90	99	108
=10	10	20	30	40	50	60	70	80	90	100	110	120
=11	11	22	33	44	55	66	77	88	99	110	121	132
=12	12	24	36	48	60	72	84	96	108	120	132	144

		1x	2x	3x	4x	5x	6x
1x	1	2	3	4	5	6	7
2x	2	4	6	8	10	12	14
3x	3	6	9	12	15	18	21
4x	4	8	12	16	20	24	28
5x	5	10	15	20	25	30	35
6x	6	12	18	24	30	36	42
7x	7	14	21	28	35	42	49
8x	8	16	24	32	40	48	56
9x	9	18	27	36	45	54	63
10x	10	20	30	40	50	60	70
11x	11	22	33	44	55	66	77
12x	12	24	36	48	60	72	84

Procedural knowledge

The key methods are **short division** and **long division**.



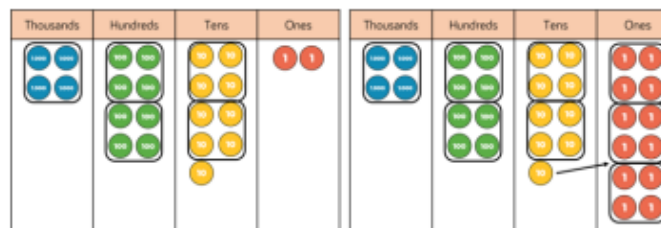
	0	4	8	9	
15	7	3	3	5	
-	6	0	0	0	(x400)
	1	3	3	5	
-	1	2	0	0	(x80)
		1	3	5	
-		1	3	5	(x9)
				0	

Key vocabulary: quotient, divisor, dividend, divide, repeated subtraction

Division in Year 6

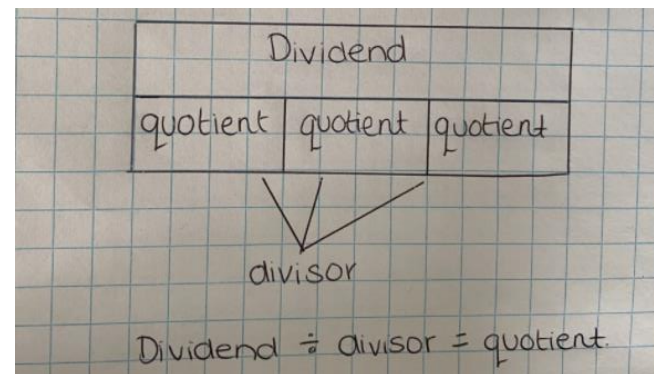
1

1. The recommended manipulatives (physical resources) for dividing 4- digit numbers by 2- digit numbers are **place value counters and dienes**.



2

2. The key representations used are: **blank number lines** (to show the link with repeated addition), **and bar model**.



3

3. The key methods are **short division and long division**.

	1	2	2	3		
4	4	8	9	4		r2

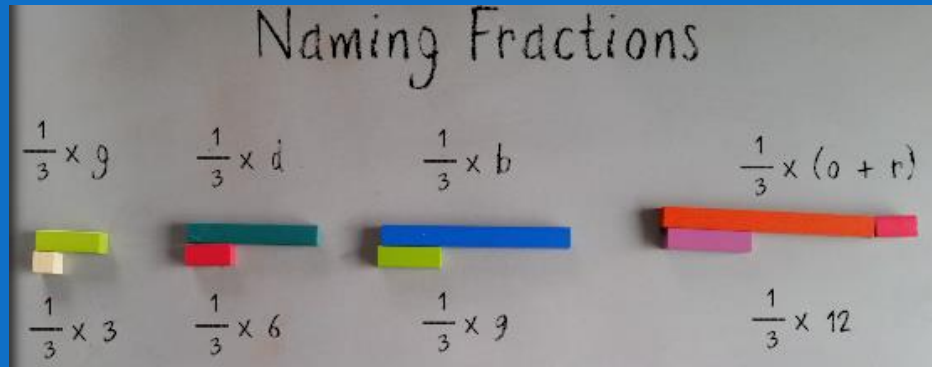
	0	4	8	9	
15	7	3	3	5	
-	6	0	0	0	(x400)
	1	3	3	5	
-	1	2	0	0	(x80)
		1	3	5	
-		1	3	5	(x9)
				0	



YEAR 6: Fractions

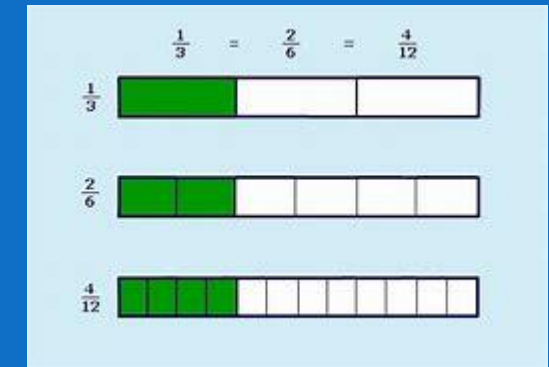
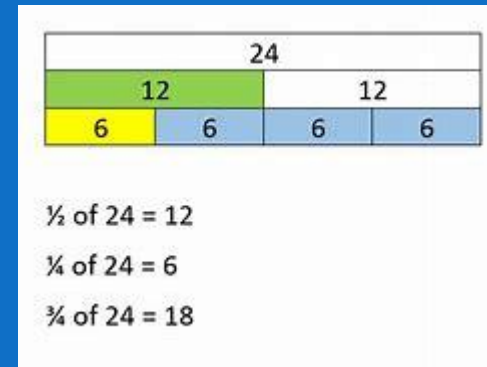
Manipulatives

The recommended manipulatives (physical resources) for fractions are **Cuisenaire**.



Representations

The key representations are **number lines**, **PPW diagrams** and **bar models**.

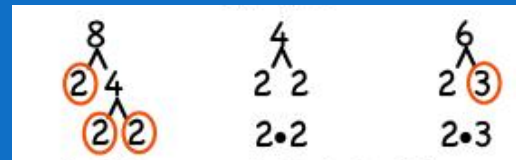


Factual knowledge

The key factual knowledge includes the recall and recognition of **equivalent fractions** using common multiples (generation) and common factors (simplification), recall common **FDP equivalences** and associate a fraction with the **division equation**.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

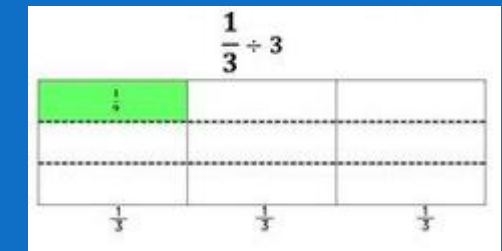
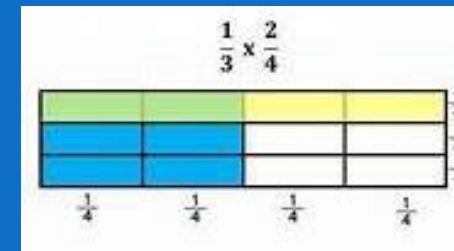
YOU CAN READ EQUIVALENT FRACTIONS ALONG EACH ROW OF THE MULTIPLICATION CHART! A FEW EXAMPLES ARE HIGHLIGHTED, BUT IT WORKS WITH THE ENTIRE CHART!



If $\frac{1}{2} = 1 \div 2$ then $\frac{1}{4} = 1 \div 4$

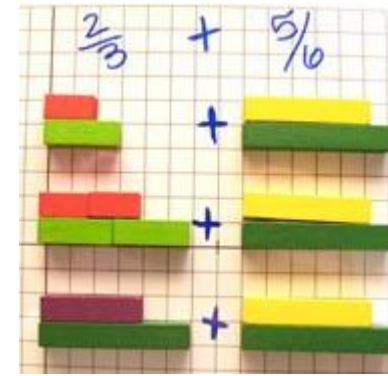
Procedural knowledge

The key procedures are **ordering** fractions >1 , **adding/subtracting** fractions including mixed numbers, **multiplying** pairs of proper fractions, **dividing** fractions by an integer.



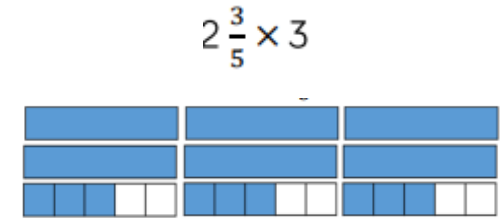
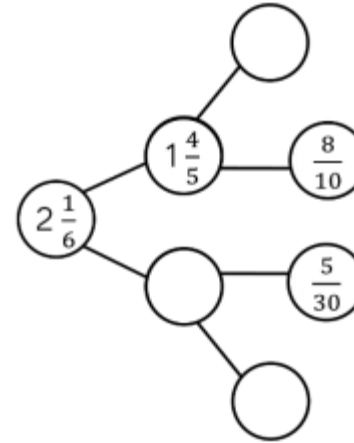
Fractions in Year 6

1. The recommended manipulatives (physical resources) for fractions are **two-colour counters** and **Cuisenaire rods**.



2. The key representations are **blank number lines**, **part-part-whole diagrams** and **bar models**.

2



3. The key procedures are **ordering** fractions >1 , **adding/subtracting** fractions including mixed numbers, **multiplying pairs of proper fractions**, **dividing fractions** by an integer.

3

